# Capturing Students' Thinking about Strategies used to Solve Mental Computations by Giving Students Access to a Pedagogical Framework


Judy Hartnett
*Brisbane Catholic Education*
*<jhartnett@bne.catholic.edu.au>*



Capturing evidence of the strategies that students use when completing mental computation is difficult to do in everyday classroom practice. Teachers do not have access to or time to analyse verbal protocols as researchers do. This study shows that meaningful written recordings of strategy choice and student thinking while completing computations are possible when a scaffolded framework for teaching the strategies is used and the students are given access to this pedagogical framework in a way they can understand and apply.


Goals and content of school mathematics are changing and the change is not restricted to Australia. The focus is not only on the delivery of content knowledge, skills and procedures but also on assisting students to develop deeper understanding of mathematics concepts and the processes; for example, thinking, applying, communicating and reflecting. The National Statement on Mathematics for Australian Schools (Australian Education Council, 1991) which is still the most recent "policy" document about mathematics education in Australia, stated that "learning mathematics involves both its products (body of knowledge) and processes (ways of knowing)" (p. 26). The new Queensland Essential Learnings framework describes "the knowledge, understanding and ways of working that students need for ongoing learning, social and personal competence and participation in a democratic society" (Queensland Studies Authority, 2008). This aligns with Skemp's (1976) description of relational understanding (How does this make sense with in relation to what I understand?) as opposed to instrumental understanding (What do I have to do?).

## Recording Student Thinking

To make judgements about student learning of mathematics, teachers need to consider both aspects of mathematical learning – knowledge and processes. It is easier for teachers to focus on the mathematical knowledge and skills aspect of learning that are visible and can be evidenced through observation, focussed analysis, checklists etc. Evidencing thinking and ways of working is more difficult. In research situations the use of individual student interviews with video or audio recording can assist in capturing evidence of student thinking and processes. Techniques like asking students to "think-aloud" to explain strategies used or what students understand about a concept can be utilised to structure the data collected. Teachers do not have easy access to these techniques in their everyday practice, nor do they have time to analyse them closely so they tend to rely on written evidence in the form of checklists of observed behaviours, student work samples, written tests etc. Also these records can be collected in the form of a student folio and reflected on for reporting to parents and to evidence progression of learning. Evidence of thinking, strategy choice and conceptual understanding can be recorded as anecdotal records based on observation or discussions with the student. Often these assessments are seen as subjective and not "hard evidence". A method for collecting written forms of students thinking and strategy choices that does not require transcribing recorded speech would be a useful tool for teachers' assessment of mathematical learning in their classrooms.

Researchers have considered student written recordings as a way of capturing student thinking. Rose (1989) described writing as a valid way of "thinking aloud on paper". Pugalee (2004) compared verbal and written descriptions of students involved in problem solving activities and described situations where students were asked "to record any working for the problem on the paper provided….". One group was specifically asked to "write everything which comes to mind during the solving of the problem" while the other group were told "please think out loud by telling everything that comes to mind while you are solving the problem" (p. 33). He found that the strategies used by students did not vary greatly between those who provided written or verbal descriptions of the problem solving processes. The students who wrote about their processes produced correct solutions at a statistically higher rate than those using the think-aloud method.

Vygotsky (1987) described writing as involving deliberate analytical action on the part of the writer requiring the writer to maximally compact inner speech so that it is fully understandable. He also viewed writing as important in forming associations between current and new knowledge, helping the writer organise ideas

Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia
M. Goos, R. Brown, & K. Makar (Eds.), © MERGA Inc. 2008

in order to make connections between prior and new concepts. Research has shown that writing provided a level of reflection that promoted students' attention to their thinking about mathematical processes (Carr & Biddlecomb, 1998; Pugalee, 2001). Pugalee (2004) stated that this awareness and self regulation appeared to play an important part in students' selection of appropriate information and strategies.

## Effective Mental Computation

The focus of computation instruction has shifted from developing students' proficiency with the traditional written algorithms to a focus on strategy use and development of number sense. Research into identification of effective mental computators has taken different approaches to the identification of successful students. Many early studies equated success with speed and accuracy only. A common research method was timed tests (Reys, Reys, & Hope, 1993). These studies were unable to identify strategies used by the students as students were asked to write down answers only after calculating mentally. Their thinking and processes were not able to be captured by the researchers. It has been noted that accuracy by itself is not sufficient as a model for successful mental computation (Heirdsfield, 2001; Thompson, 1999). In some studies, for example, Hope and Sherrill (1987), this type of testing was used in conjunction with further interviews to identify student strategies that gave information on more than one component of mental computation.

Heirdsfield (2001) concluded that mental computation is calculating using strategies with understanding, and thus, proficiency in mental computation was not confined to accuracy, but also included flexibility of strategy choice. Thus successful mental computators need a variety of strategies with which they are comfortable and understand the application of, as well as flexibility to choose from known strategies according to the problem context.

To enable teachers to judge how effective students are in regard to mental computation their thinking about strategy choice and application needs to be captured as well as their answers. Panaoura and Philippou (2005) noted that asking young students about their cognitive processes involves some particular problems. Young students have limited experience with the world and limited vocabulary on which to draw, and as such their experience with certain maths concepts is limited to what they are able to articulate. Panaoura and Philippou supposed that children's answers may reflect not what they know or believe, but rather what they can or cannot tell to the interviewer.

There have been studies where young students verbalised strategies used for computation but few studies involving young students recording their thinking in writing. McIntosh (2002) developed informal written recording processes for mental computation, which showed that it was possible for primary school students to record their strategies.

Asking students to record responses to computation questions in writing does not automatically elicit their thinking strategies. Asking students to show their thinking on paper could elicit the traditional written algorithms rather than the targeted thinking and strategy choice used for mental computation. Younger students are still developing their writing skills and knowledge of mathematical symbols and as such their recording methods may not capture the metacognitive processes they are using. Scaffolding the process of strategy instruction, discussion and recording thinking could assist these students to be able to record their thinking and choice of strategy. This would enable teachers to "see" students' thinking and make assessment judgements not only about the content they know and the types of problems they can solve, but how they are doing this and what they do or do not understand about computation concepts and number sense.

## A Framework for Mental Computation Strategies

Hartnett (2007) proposed a categorisation framework of mental computation strategies. The intention of the strategy categorisation was to create a small number of general categories with intuitive labels that would make sense to teachers and also to the students. A list of sub-categories made clearer the variations that could be a focus in each category. In all, five major categories and 21 sub-categories were identified (see Table 1). With the labels for the categories kept in simple language it was intended that these would be used in the classroom as the focus of lessons and to facilitate the discussion of strategies used by students. By presenting a coherent way of thinking about the possible mental computation strategies the teacher and the students would have a common language for discussions about strategies. This framework provides structure learning for activities, student thinking and strategy choice and also a structure for the recording of student thinking during computation activities and assessments.

## Method

A focus class was chosen from a suburban Catholic primary school in Brisbane, Australia. Teachers at this school had a shown an interest in the development of mental computation strategies. The subjects were one class of 27 Year 3 students (8-9 year olds). A Year 3 class was chosen as the focus class due to this being the year when computation, particularly on two digit numbers, is a major topic of learning. The students had not had any exposure to the traditional written algorithms that are commonly introduced in this year level and often before.

The researcher planned for and taught one strategy development lesson each week based on Hartnett's (2007) strategy categories (see Table 1), and the classroom teacher followed this with further lessons during the week. The strategies were explained and referred to consistently by their category and sub-category names; for example, a series of lessons focussed on teaching the "breaking up two numbers using place value" strategy. The researcher and the class teacher modelled the recording of thinking during lessons. They also provided and modelled the use of structures for supporting computation strategies, for example, number boards, empty number lines, arrows indicating progression of thinking etc.

**Table 1**

*Categorisation of Mental Computation Strategies (Hartnett, 2007)*

| Strategy category | Strategy sub-categories |
|---|---|
| Count On and Back | ▪ Count on to add |
| | ▪ Count back to subtract |
| | ▪ Count on to subtract |
| | ▪ Count on to multiply |
| Adjust and Compensate (Change and Fix) | ▪ Adjust one number and compensate |
| | ▪ Adjust two numbers and compensate |
| | ▪ Adjust two numbers |
| Break Up Numbers | ▪ Break up two numbers using place value |
| | ▪ Break up two numbers using compatible nos. |
| | ▪ Break up one number using place value |
| | ▪ Break up one number using compatible nos. |
| Double and /or Halve | ▪ Use a double or near double to add or subtract |
| | ▪ Double to multiply by 2 |
| | ▪ Double, double to multiply by 4 |
| | ▪ Double, double, double to multiply by 8 |
| | ▪ Half to divide by 2 |
| | ▪ Half, half to divide by 4 |
| | ▪ Half, half, half to divide by 8 |
| | ▪ Double and halve |
| Use Place Value | ▪ Think in multiples of ten |
| | ▪ Focus on relevant places |

Much of the research on strategy use in computation has focussed on students working with examples involving small numbers (to 20). In this study a deliberate focus was made on using larger numbers that were appropriate as a computational instruction focus for this year level. The focus operations were addition and subtraction on two and three digit numbers as outlined in the current Mathematics syllabus (Queensland Studies Authority, 2004). A variety of number combinations, including those requiring the bridging of ten (e.g., 19+12 and 100–36), were included as they suited a strategy focus for computation. They also provided a change in focus for teaching as traditionally in Year 3 examples requiring regrouping were left until after students could complete examples without regrouping when being taught the traditional written algorithms.

The students completed a pre- and post-test of a range of computation questions chosen to allow for a range of strategies to be used across both single and two-digit examples at the beginning and the end of a school year. They also completed mid-year assessments that included some items from the pre-test and some others. For this paper a comparison of the pre- and post-tests looking for evidence of recording of strategy choice, including use of the category labels, was the focus. The students were asked in both the pre- and post-tests to record their thinking so that someone reading their response would understand how they had worked out their answer.

To determine how the students had described the strategies they used their descriptions in the pre- and post-tests were reviewed for a change in the number of students who were able to clearly describe the strategy that they had used from the pre- to the post-tests. Special note was taken of students who used the actual strategy category names from the framework.

The class teacher was interviewed informally throughout the study and the researcher kept field notes of these discussions. She was also interviewed formally at the end of the study.

## Results and Discussion

The students in the class chose to alter the "Adjust and Compensate" category to call it "Change and Fix" which they thought was a better description of the strategy. This showed that they felt comfortable with the labels and had a deeper understanding of their usage than just remembering the name.

In the pre-test many students only recorded an answer (58.2%). This was likely to be due to inexperience with this way of working and/or lack of the language to describe them. There was no way to deduce which strategy or strategies a student had utilised. In the pre-test 30% of the students made no response to pre-test questions and 12.5% of the students attempted to record their thinking. Of the students who attempted to record their thinking only 2% of these managed to make the explanation of their strategies clear. In the post-test 87% of the students made an attempt to record their strategies with 63% of these doing so in a way that was clearly understood. Although the number of students whose strategy recording was unclear also rose, along with the increase in attempted recordings this shows that a large proportion of the students were growing in confidence with their ability to record their thinking and strategies used.

Figure 1 summarises the change in the percentages for each type of response in the pre- and post-tests for the focus class.

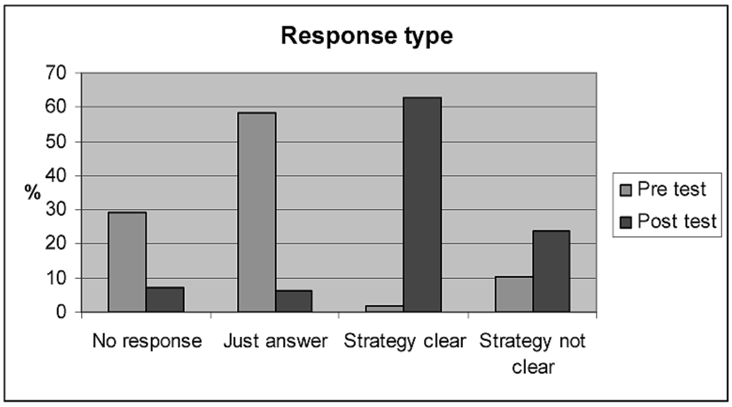


*Figure1.* Percentage of students who used each type of response in the pre- and post-tests.

The number of students who just provided the answer reduced from 58% in the pre-test to 6% in the post-test. The questions that elicited just responses in the post-test tended to be ones referred to as retrieval strategies (Siegler, 1987) and the students recorded "I just knew it" or similar beside their answer. Siegler (1988) noted that more knowledgeable students tend to use retrieval more often and to answer more quickly and accurately. This was the case in this study where the students who exhibited such responses tended to be the more capable students, identified by consistently high success with the questions on all tests and who had been identified as above average by the class teacher. For this study students who wrote such responses were categorised as providing just answers.

The descriptions that the students used to explain their strategies in the post-test were analysed according to whether they used the actual name of the strategy from the framework (see Figure 2); whether they used a method of showing the strategy used that had been demonstrated during the lessons, for example, empty number line (see Figure 3); or whether the strategy used was evident through a personal explanation that allowed the teacher to determine the strategy (see Figure 4).


*Figure 2.* Student use of categorisation framework labels.

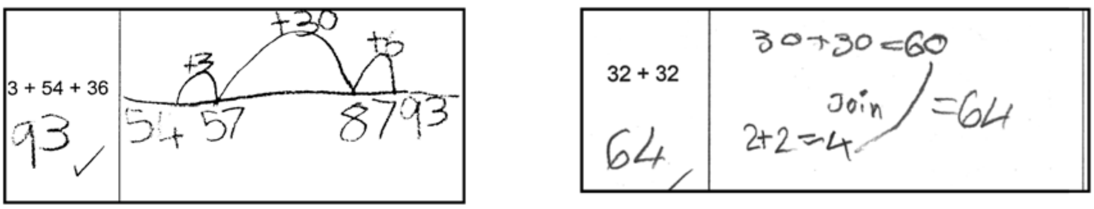


*Figure 3.* Use of recording matching as demonstrated or discussed during lessons.

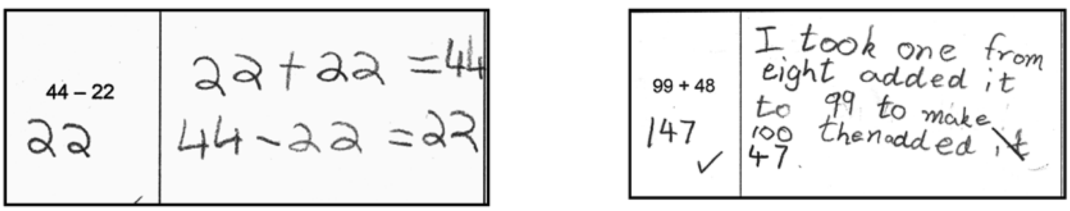


*Figure 4.* Student’s own description of the strategy used.

Table 2 shows the percentage of students across all the questions in the post test who showed each of these descriptions. The remaining percentage was students who gave no response or just an answer.

**Table 2**

*Variations in Strategy Descriptions by Focus Class Students*

| | Use of strategy categorisation labels | Strategy as demonstrated in lessons (without label) | Strategy obvious - student used own descriptions |
|---|---|---|---|
| Post-test | 12.35% | 63.43% | 10.49% |

## Conclusions

The aim of this study was to trial the use of ways to capture the strategies used by students when completing mental computation questions. The results show that it is possible for young students to record their thinking and the strategies used so others can understand what they did. The use of the categorisation framework (Hartnett 2007) provided a structure for the teacher to organise learning activities for the students providing comprehensive coverage of a range of strategies appropriate to the numbers that were the focus of computation for this year level. The framework labels provided a structure for classroom dialogue about strategies used and the students were able to show this in their recording of their thinking. Over 75% of post-test strategy descriptions used the framework labels or strategy applications that had been demonstrated in lessons. The use of the strategy framework labels was never a requirement for the students but the number of the students who used them without prompting shows an understanding of the framework and that they must have felt this would help explain what they had done.

The teacher was pleased to have "hard" evidence of the students' understandings that she was able to keep and refer to for planning further lessons, discussing with the students, making assessment judgements, and reporting to parents. She was able to see development of strategies and concepts across the year and was able to identify misunderstandings in the methods some of the students were using. An example of this remedial use of the student descriptions was when some students had recorded just an answer of 10 for 32–18 in the mid-year test. The strategy recording provided by one student (See Figure 5), allowed the teacher to identify the misconception. The students had been wrongly applying the "Breaking up two numbers using place value" strategy in the subtraction. After discussion with these students and consequently the whole class, the error was clarified and deeper understanding of the potential difficulties with the application of this strategy particularly for subtraction was discussed and alternative strategies were proposed.

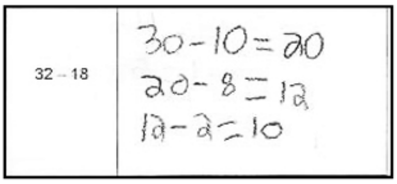


*Figure 5.* Evidence of a misconception about a strategy in a student's recording.

In this study, the students were given access to knowledge that is usually provided only for teachers – namely the strategy categories. They coped very well with this and the teacher commented that she thought they seemed "proud of themselves" to be able to use what they considered teacher talk with her and the researcher.

## Implications for Further Research

The study was only for one year and as such there were limitations in the internalisation of the strategies by the students. The students were really only beginning to gain familiarisation with computation with two-digit and larger numbers as well as with the framework. It would be interesting to follow students who have worked with the strategy categories for all of primary school noting differences in strategy use and thinking as evidenced through their recordings across all operations and with other numbers, for example, decimals.

The school involved has contracted to work with the researcher to develop a whole school approach to teaching computation strategies based on this work. With further scaffolding of strategies in future years it is hoped that more of the students would be likely to develop proficiency at justifying their thinking and communicating their methods in writing thus providing assessment data of this nature for the teacher.

The use of the strategy category framework to further examine communication about thinking and to guide student recording of their thinking and strategies could be examined in relation to student to student communication.

The recording of thinking in this study focussed on capturing computation strategies. There is scope for the capturing of other thinking strategies like those used with problem solving strategies especially if a framework for these strategies was part of the instruction.

# References


Australian Education Council (AEC). (1991). *A national statement on mathematics for Australian schools*. Melbourne: Curriculum Corporation.

Carr, M., & Biddlecomb, B. (1998). Metacognition in mathematics: From a constructivist perspective. In D.J. Hacker, J. Dunlosky, & A.C. Graesser (Eds.), *Metacognition in educational theory and practice*. Mahwah, NJ: Lawrence Erlbaum Associates.

Hartnett, J. E. (2007). Categorisation of mental computation strategies to support teaching and to encourage classroom dialogue. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 345-352). Sydney: MERGA.

Hatano, G. (1988). Social and motivational bases for mathematical understanding. *New directions for child and adolescent development*, *41,* 55-70.

Heirdsfield, A. M. (2001). *Mental computation: The identification of associated cognitive, metacognitive and affective factors.* Unpublished doctoral dissertation. Queensland University of Technology.

Hope, J. A., & Sherrill, J. (1987). Characteristics of unskilled and skilled mental calculators. *Journal for Research in Mathematics Education, 18*(2), 98-111.

McIntosh, A. (2002). Developing informal written computation. In M. Goos & T. Spencer (Eds.), *Mathematics making waves* (Proceedings of the 19th biennial conference of the Australian Association of Mathematics Teachers, Brisbane, pp. 376-383). Adelaide: AAMT.

Panaoura, A. & Phillippou, G. (2005) *The measurement of young pupils' metacognitive ability in mathematics: The case of self-representation and self-evaluation.* Retrieved 6 February 2008 from http://www.cerme4.crm.es/papers%20definitus/2/panoura.philippou.

Pugalee, D. K. (2001). Writing, mathematics and metacognition: Looking for connections through students' work in mathematical problem solving. *School Science and Mathematics*, *101*(5)*,* 236-245.

Pugalee, D. K. (2004). A comparison of verbal and written descriptions of students' problem solving processes. *Educational Studies in Mathematics*, *55*, 27-47.

Queensland Studies Authority. (2004). *Mathematics years 1 to 10 syllabus.* Brisbane: QSA.

Queensland Studies Authority. (2008). *Queensland curriculum, assessment and reporting (QCAR) framework*. Retrieved 6 February 2008 from http://www.qsa.qld.edu.au/assessment/qcar.html.

Reys, R. E., Reys, B. J., & Hope, J. A. (1993). Mental computation: A snapshot of second, fifth and seventh grade performance. *School Science and Mathematics, 93*(6), 306-315.

Rose, B. J. (1989). Writing and math: Theory and practice. In P.Connolly & T.Vilardi, (Eds.), *Writing to learn mathematics and science.* New York: Teachers College Press.

Siegler, R. S. (1987). Strategy choices in subtraction. In J. Sloboda & D. Rogers (Eds.), *Cognitive process in mathematics* (pp. 81-106). Oxford: Oxford University Press.

Siegler, R. S. (1988). Strategy choice procedures and the development of multiplication skills. *Journal of Experimental Psychology: General*, *117,* 258-275.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77*, 20-26.

Vygotsky, L. S. (1987). Thinking and speech. In R. W Rieber & A. S. Carton (Eds.), *The collected works of L.S. Vygotsky* (pp. 39-243). New York: Plenum Press.